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How Many Small Firms are Enough?

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Abstract

Individual investors are typically undiversified, holding on average less than four securities in their personal portfolios. Because the small firm literature has focused upon CAPM (systematic risk, full diversification concept) premia, the actual performance of small firm portfolios which are actually held by investors may be seriously overstated because of the presence of unsystematic risk. Therefore, a question that is of interest is at what point in the diversification process does the "small firm effect" take hold? The empirical results in this paper illustrate the magnitudes of total risk for small and large firms as well as the behavior of such measures as portfolio size is altered. That small firms contain more risk is shown by the observation that a market portfolio of small firms has greater variability than a single, typical large firm. While these results indicate the superiority of small firms, investors should be aware of the return performance implication of small portfolios. Because small firms contain such extreme amounts of unsystematic risk, diversification is essential if investors are going to capture the small firm premia reported in the literature.

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How Many Small Firms Are Enough?

I. Introduction

The findings in the research of Banz (1981) and Reinganum (1981a) have spawned considerable interest into why and if small firms earn superior systematic risk-adjusted returns relative to large firms. Although some are still skeptical because of the special problems involved with measuring the systematic risk (see Dimson, 1979; Fowler and Rorke, 1983; Roll, 1981; and Scholes and Williams, 1977) as well as the returns (Roll, 1983; Blume and Stambaugh, 1983) of small firms, by and large the conclusion seems to be that small firms do warrant investor attention.

In research of a different nature, Blume and Friend (1975) found in their investigation of investor holdings that individual investors are typically undiversified, holding on average less than four securities in their personal portfolios. Theoretical work by Levy (1978) and Mayshar (1979, 1981) has examined equilibrium conditions under the assumption that investors are not diversified. Of particular interest in this research is the finding that total risk (variance) is more important than systematic risk (beta) in the pricing of assets, particularly for small firms (emphasis added).

Because the small firm literature has focused upon CAPM (systematic risk, full diversification concept) premia, the actual performance of small firm portfolios which are actually held by investors may be seriously overstated because of the presence of unsystematic risk. Therefore, a question that is of interest is at what point in the diversification process does the "small firm effect" take hold?

The purpose of this research is to examine the behavior of the small firm premia--the excess risk-adjusted performance of small firms relative to large firms--in response to changes in portfolio size. Using security data, this issue is examined for three non-overlapping five year periods. The results indicate the presence of a small firm premia even at small portfolio sizes. But, because small firms contain extreme amounts of unsystematic risk relative to large firms, the magnitude of the small firm premia for small portfolios is considerably smaller than the systematic premia reported in the literature. Thus, for investors with few security holdings, a large portion of the systematic small firm premia remains to be captured.

The following section develops the diversification properties of portfolio risk and the small firm premia, while Section III describes the data base and methodology. Section IV presents the empirical results and Section V contains conclusions regarding diversification in small firm portfolios.

II. Diversification, Risk and the Small Firm Premia

A. The Measurement of Portfolio Risk

Traditionally, security risk has been measured on an ex post basis by the variability in the time series of returns. In a portfolio context, this variance in return, σ_n^2 , on a portfolio of n securities is:

$$\sigma_n^2 = \sum_{i=1}^n \sum_{j=1}^n x_i x_j \sigma_{ij} \quad (1)$$

where x_i and x_j represent portfolio proportions invested in securities i and j and σ_{ij} indicates the covariance between the time series returns on i and j .

Under a policy of naive or random diversification (an equal investment in each portfolio security), $x_i = 1/n$. The impact of diversification on σ_n^2 can be evaluated analytically by decomposing equation (1) into its components and expressing expected portfolio risk as (see Elton and Gruber, 1977, equation (B3)):

$$E(\sigma_n^2) = [(1/n)\bar{\sigma}^2(1-(n-1)/(N-1))] + [(n-1)N\sigma_N^2/n(N-1)] \quad (2)$$

where: n = number of securities in the portfolio

N = number of securities in the population

$\bar{\sigma}^2$ = average variance for a one security portfolio

σ_N^2 = market (systematic) risk for an equally weighted portfolio of all N securities in the population.

Equations (1) and (2) indicate that the extent to which diversification will reduce portfolio risk to its systematic level σ_N^2 (when $n=N$, $E(\sigma_n^2) = \sigma_N^2$) is a function of the correlation relationships present in security return distributions. More importantly, equation (2) provides an analytical (exact) expression of portfolio variance for any portfolio size for any security population. Once the average individual security variance, $\bar{\sigma}^2$, and the market variance, σ_N^2 , have been determined, equation (2) can be used to calculate the expected (average) portfolio risk for any portfolio size. Furthermore, expressing portfolio risk in this manner avoids the interpretational problems associated with using risk measures computed from the betas derived from alternative market indexes (Roll, 1977, 1981, 1983).

B. Measuring the Small Firm Premia

Prior analyses of small firms by Reinganum (1981b, 1982, 1983a), Roll (1981, 1983) and others have evaluated the importance of the performance of small firms through a comparison of the excess (in excess of the risk-free rate, r_f) systematic risk (beta) adjusted return on the small (typically the bottom decile in a market valuation ranking) firm portfolio relative to that of the large (top decile) firm portfolio. Since the comparison is always made between large, well-diversified portfolios, the portfolio variance equivalent of this premia is:

$$SF_N = [E(r)_{N,S} - r_f]/\sigma_{N,S} - [E(r)_{N,L} - r_f]/\sigma_{N,L} \quad (3)$$

where: SF_N = systematic small firm premia for a market portfolio of N securities

$E(r)_{N,S}$, $E(r)_{N,L}$ = expected returns for market portfolios of small (S) and large (L) firms

$\sigma_{N,S}$, $\sigma_{N,L}$ = the standard deviations of the two respective portfolios.

However, measuring the premia through a comparison of the performances of two well-diversified portfolios assumes that investors have diversified the unsystematic risks present within the two respective portfolios. Of interest to individual, undiversified investors is the magnitude of the following premia:

$$SF_n = [E(r)_{n,S} - r_f]/\sigma_{n,S} - [E(r)_{n,L} - r_f]/\sigma_{n,L} \quad (4)$$

where SF_n = the small firm premia at portfolio size n

Because equation (4) measures the relative performance at diversification levels smaller than N , it explicitly considers the presence of unsystematic risk in the portfolios. The directional change in SF_n with increasing n will depend upon the relative magnitudes of unsystematic risk in small and large firm portfolios. If small firms contain more unsystematic risk, then SF_n will increase with increasing n . Furthermore, the magnitude and sign of SF_n at any particular portfolio size n will depend not only upon the relative levels of risk (systematic plus unsystematic) for small and large firms, but also upon the relationships between excess returns and total risks for individual portfolios within the two size structures, a matter which can only be resolved empirically. If the additional (unsystematic) risk of undiversified portfolios of small firms exceeds that of comparably sized portfolios of large firms, SF_n can be negative even though SF_N is positive. Having discussed these considerations, the effects of portfolio size upon the portfolio risks of small and large firms as well as the small firm premia are now empirically examined.

III. The Data and Methodology

Three non-overlapping five year periods are chosen to illustrate the effects of diversification upon small and large firm unsystematic risks and the small firm premia. The periods include: January, 1967-December, 1971; January, 1972-December, 1976; and January, 1977-December, 1981. These three five year periods were chosen to examine the stationarity of the relationships over time and to produce periods

of sufficient duration so as to provide meaningful estimates of individual security as well as portfolio total risks. Each of the periods contains a complete market cycle marked by at least one downturn in the stock market.

At the beginning of each five year period, firms were rank ordered according to market values (number of shares outstanding x market price per share) and the bottom (smallest) and top (largest) deciles were extracted for analysis. The investment characteristics of these six samples are shown in Table 1. As shown in Table 1, the small (large) firm samples are almost exclusively AMEX (NYSE) firms. While large firms, on average, grew in all periods, the typical small firm actually declined in value during the 1972-1976 period, indicating the volatility of such issues.

For each of the three periods, individual security and portfolio return and risk measures are computed in the following manner. First, the daily returns on each stock i ($i=1, \dots, N$) for each month t ($t=1, \dots, 60$) are compounded to produce a monthly return:

$$r_{i,t} = [\pi(r_{i,\tau} + 1)] - 1$$

where: $r_{i,t}$ = monthly return on security i in month t

τ = return in day τ

Second, from this series the security monthly average returns and variances are then calculated:

$$E(r_i) = \bar{r}_i = 1/60 \sum_{t=1}^{60} r_{i,t}$$

$$\sigma_i^2 = 1/59 \sum_{t=1}^{60} (r_{i,t} - \bar{r}_i)^2$$

Third, the cross-sectional average security variance is computed from the individual security risks:

$$\bar{\sigma}^2 = 1/N \sum_{i=1}^N \sigma_i^2$$

Finally, monthly returns and variances are computed for the six equally weighted market portfolios:

$$r_{N,t} = 1/N \sum_{i=1}^N r_{i,t}$$

$$\bar{r}_N = 1/60 \sum_{t=1}^{60} r_{N,t}$$

$$\sigma_N^2 = 1/59 \sum_{t=1}^{60} (r_{N,t} - \bar{r}_N)^2$$

With regard to this methodology, several comments are in order. First, returns are reviewed monthly. For portfolios, this implies that portfolio weights are rebalanced at the end of the month (as opposed to daily) and the portfolios are held for five years. This methodology can be classified as the "buy and hold" approach of computing returns - where individual security daily returns are first compounded into monthly returns and then portfolio monthly returns are equally weighted security monthly returns. According to Roll (1983), the "buy and hold" method best mimics actual investment performance and produces less bias in computing returns than alternative methods which

first average security daily returns, then compound the daily portfolio returns to produce a portfolio monthly return.

Second, a five year examination period enables the use of monthly returns which alleviates much of the bias present in risk measures computed from daily data (non-synchronous trading problem). Because returns calculated via the buy and hold method are primarily affected by individual asset return dependencies (Roll, 1983), it is instructive to examine the serial correlation present in the securities' returns. As shown in Table 2, there is some degree of auto-correlation among security monthly returns. However, given the sizes of the security samples, it does not appear to be a major problem.

IV. The Results

The objectives of the empirical analysis are to illustrate the relative amounts of risk present in small and large firms and to examine the effects of diversification on the investment performances of small and large firms. First, summary return and risk statistics are presented. These data allow a comparison of the levels of systematic and unsystematic risk among the samples. Next, the risk adjusted performances of small and large firms are shown to examine the effects of diversification upon these measures.

A. Return Distribution Statistics for Alternative Small and Large Firm Samples

Table 3 presents return distribution statistics for the six security groups examined. Line 1 reveals that while the average monthly return, \bar{r}_N , across large firms is fairly constant across the periods,

average returns for small firms fluctuated considerably across the periods, ranging from 1.132% in 1972-1976 to 2.89% in 1977-1981.

Interestingly, the average risk statistics (line 2) indicate fairly stable levels for both small and large firms. The first (1967-1971) and third (1977-1981) period values are especially close and while the second period values are higher, this can be attributed primarily to the severe market downturn in 1974. The 1974 recession is also reflected by the low average return for small firms during this period. The differences in market variances, σ_N^2 , between the periods reflects the fact that intra-sample correlations are declining over the fifteen year period (see the last line of Table 4). That is, even though the σ^2 figures for both small and large firms are essentially the same in the first and third period, the latter period's market risks are smaller due to less inter-relationships among the component securities.

The excess return/risk measures for the securities as well as their market portfolios are given in lines 4 and 5 and demonstrate a wide variety of values. The second period results reflect the impact of higher risk levels upon steady (large firms) and falling (small firms) return levels. The negative performance of large firms in the last period indicates the dramatic increase in Treasury bill yields (line 6) relative to large firm returns (line 1). But, of particular interest is the finding that small firms outperformed large firms throughout the entire fifteen year period. Since investors can diversify their holdings to modify portfolio return distribution characteristics, it is instructive to examine the empirical behavior of portfolio risks for small and large firms in response to changes in portfolio size.

B. Diversification and Small and Large Firm Portfolio Risk

Expected (average) values of portfolio risk at any portfolio size can be determined using equation (2) and the sample data in Table 3. These measures are presented in Table 4. Table 4 shows that diversification dramatically reduces the level of risk, particularly for small firms. The last line of each column of Table 4 reveals that not only are small firms less correlated than large firms, but this relative relationship has widened over the past fifteen years. In 1967-1971, both types of firms had about 65 percent of their total risk which was diversifiable; in 1977-1981, small and large firms had about 76 and 71 percent, respectively. The last line also indicates a decline in the relative amounts (relative to total risk) of systematic risks for both small and large firms indicating the increasing benefits from diversification.

However, due to the differences in magnitudes of the numbers, the results presented in Tables 4 illustrate that the traditional method of analyzing diversification benefits in terms of percentages can be misleading when applied to the small-large firm distinction. First, small and large firms differ significantly in the magnitudes of their total and systematic risk. The relative magnitudes of total or systematic risk in small firms are on the order of four to five times the respective levels found in large firms. Second, for the periods examined in this research, a small firm portfolio of any size has more risk than the typical single large firm. That is, the σ_N^2 of small firms exceeds $\bar{\sigma}^2$ for large firms in all three periods. This is a lot of risk, when you consider, for example, that during 1977-1981 the small

firm market portfolio has 160 firms whose joint variability (σ_N^2) of 62.962 exceeds the typical total risk ($\bar{\sigma}^2$) for a large single firm of 55.215.

C. Diversification and the Small Firm Premia

Because the excess return/risk, $(\bar{r}_n - \bar{r}_f)/\sigma_n$, represents the ratio of two random variables, there is no exact analytical expression for its value at a particular portfolio size (Mood, Graybill and Boes, 1974). Consequently, simulation is necessary to investigate the impact upon the performances of small and large firms. For this purpose, one thousand portfolios are selected randomly with replacement, at $n = 2, \dots, 5, 10, 20, \dots, 50$. For each portfolio of size n , the above time series performance measure is computed and then averaged to provide the average value of $(\bar{r}_n - \bar{r}_f)/\sigma_n$ at each portfolio level for each sample. Size one and market values of the ratio are computed directly from the data. These values are presented in Table 5.

As expected, the average $(\bar{r}_n - \bar{r}_f)/\sigma_n$ increases with diversification for all groups since portfolio expected return is constant, but portfolio risk declines as n increases. As the table indicates, small firms outperform large firms across the periods.

Of particular interest is the relationship between SF_n , the small firm premia at portfolio size n , and SF_N , the systematic risk small firm premia as n changes. As seen in Table 5, only about 50 percent of the systematic premia is realized by holding one small firm. Because small firms contain so much unsystematic risk relative to large firms, this percentage will rise and even at portfolio size 10 only about 90 percent is realized. Furthermore, because most of the change in the

premium is due to changes in the performance of small firms, investors in small firms are advised that only with further diversification can the large amounts of small firm unsystematic risk be reduced to produce a level of risk commensurate with the risk of a large portfolio of small firms.

V. Summary and Conclusions

The empirical results illustrate the magnitudes of risk for small and large firms as well as the behavior of such measures as portfolio size is altered. That small firms contain more risk is shown by the observation that a market portfolio of small firms has greater variability than a single, typical large firm. While the results indicate the superiority of small firms, investors should be aware of the return performance implication of small portfolios. Because small firms contain such extreme amounts of unsystematic risk, diversification is essential if investors are going to capture the "small firm effect" phenomena which is reported in the literature.

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Table 1. Investment Characteristics of the Smallest and Largest Market Value Firms

	1967-1971		1972-1976		1977-1981	
	Small Firms (Bottom Decile)	Large Firms (Top Decile)	Small Firms (Bottom Decile)	Large Firms (Top Decile)	Small Firms (Bottom Decile)	Large Firms (Top Decile)
1. Number of firms in sample (N) ¹	110	110	157	157	160	160
2. Beginning median market value ²	3.31	1,332.74	4.65	1,475.99	3.96	1,917.85
3. Ending median market value ²	7.30	1,831.18	4.49	1,704.65	8.71	2,130.66
4. Percentage of firms listed on AMEX	96.4	.9	98.7	2.6	94.4	1.9

¹At the beginning of each five year period, all firms having return data over the period were rank ordered according to market valuation and the top and bottom deciles were extracted for analysis. The total number of firms for each period are: 1093(1967-1971), 1565(1972-1976), and 1595(1977-1981). The final samples of small and large firms do not include firms with missing data. The total number (including both small and large firms) excluded are: 1(1967-1971), 4(1972-1976), 17(1977-1981).

²The beginning and ending median market values (in millions of dollars) of the stocks included in each sample on the beginning and ending dates of the respective periods.

Table 2. Number of Significant Auto-Regression Coefficients and Average Auto-Regression Coefficients for the Smallest and Largest Market Value Firms¹

Lag (months)	1967-1971		1972-1976		1977-1981	
	Small Firms	Large Firms	Small Firms	Large Firms	Small Firms	Large Firms
1	2 (.042)	2 (-.041)	2 (-.095)	1 (-.043)	2 (-.095)	0 (-.024)
2	1 (-.030)	1 (-.037)	0 (-.051)	0 (-.021)	3 (-.083)	2 (-.108)
3	0 (.007)	0 (-.085)	4 (.027)	3 (.047)	1 (-.051)	0 (-.055)

¹Significant at the .01 level. Numbers in parentheses indicate the average autoregression coefficients for all stocks included in the sample, regardless of the significance level. Significance tests were also carried out to lag 10, with the results at lags 4-10 being similar to those reported here.

Table 3. Return Distribution Statistics for Alternative Samples of Small and Large Firms¹

	Symbol	1967-1971		1972-1976		1977-1981	
		Small Firms	Large Firms	Small Firms	Large Firms	Small Firms	Large Firms
1. Sample mean return	\bar{r}_N	2.310%	.679%	1.132%	.707%	2.890%	.756%
2. Average security variance	σ^2	267.459	54.602	305.049	83.710	263.605	55.215
3. Market portfolio variance	σ_N^2	91.348	19.183	98.988	29.432	62.962	15.816
4. Average security excess return/ risk ²	$E[(\bar{r}_i - \bar{r}_f)/\sigma_i]$.114	.040	.040	.030	.133	-.006
5. Market portfolio excess return/ risk	$(\bar{r}_N - \bar{r}_f)/\sigma_N$.196	.056	.066	.042	.268	-.001
6. Average monthly Treasury Bill return	\bar{r}_f	.435%		.481%		.760%	

¹ All statistics relate to monthly differencing intervals over the period examined and all values are reported in percentage terms.

² Calculated as: $E[(\bar{r}_i - \bar{r}_f)/\sigma_i] = 1/N \sum_{i=1}^N (\bar{r}_i - \bar{r}_f)/\sigma_i$

Table 4. Diversification and its Effects upon Portfolio Risk for Alternative Samples of Small and Large Firms

Portfolio size	1967-1971		1972-1976		1977-1981	
	Small Firms ¹	Large Firms ¹	Small Firms ¹	Large Firms ¹	Small Firms ¹	Large Firms ¹
1 (σ^2)	267.459	54.602	305.049	83.710	263.605	55.215
2	178.596	36.728	201.358	56.396	162.653	35.391
3	148.975	30.771	166.794	47.291	129.002	28.783
4	134.164	27.792	149.512	42.739	112.176	25.479
5	125.278	26.004	139.143	40.008	102.081	23.496
10	107.505	22.430	118.405	34.545	81.891	19.531
20	98.619	20.642	108.035	31.813	71.795	17.549
50	93.287	19.570	101.814	30.175	65.738	16.359
Market (σ_N^2)	91.348	19.183	98.988	29.432	62.962	15.816
Proportion of risk which is unsystematic:						
$1 - \sigma_N^2/\sigma^2$.658	.649	.676	.648	.761	.714

¹ Calculated using equation (2) and lines 2 and 3 of Table 3.

Table 5. Risk Adjusted Excess-Return Performances for Alternative Samples of Small and Large Firms

Portfolio Size	1967-1971			1972-1976			1977-1981			Proportion of	
	Small Firms ¹	Large Firms ¹	Systematic Small Firm Premia ² Realized	Small Firms ¹	Large Firms ¹	Systematic Small Firm Premia ² Realized	Small Firms ¹	Large Firms ¹	Systematic Small Firm Premia ² Realized	Proportion of	Systematic Small Firm Premia ² Realized
1	.114	.040	.529	.041	.030	.458	.133	-.006	.133	.517	
2	.141	.046	.678	.051	.035	.667	.167	-.005	.167	.639	
3	.156	.048	.771	.053	.036	.708	.190	-.004	.190	.721	
4	.162	.050	.800	.054	.037	.708	.202	-.003	.202	.755	
5	.171	.052	.850	.057	.038	.792	.213	-.003	.213	.803	
10	.180	.054	.900	.061	.039	.917	.238	-.002	.238	.892	
20	.188	.058	.929	.063	.040	.958	.253	-.002	.253	.948	
50	.195	.056	.993	.065	.042	.958	.263	-.001	.263	.981	
Market	.196	.056	1.000	.066	.042	1.000	.268	-.001	.268	1.000	

¹ Numbers in table represent sample average values of $(\bar{r}_i - \bar{r}_f)/\sigma_i$.

² $[E[(\bar{r}_{n,S} - \bar{r}_f)/\sigma_{n,S}] - E[(\bar{r}_{n,L} - \bar{r}_f)/\sigma_{n,L}]] / [(r_{N,S} - \bar{r}_f)/\sigma_{N,S} - (r_{N,L} - \bar{r}_f)/\sigma_{N,L}] = SF_n/SF_N$

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